Lab 1: Graphing Functions
New Functions from Old Functions

Objectives
☐ To introduce the use of LiveMath and to become familiar with its graphing capabilities
☐ To explore the behavior of families of functions.
☐ To review properties of graphs from precalculus.

Lab Exercises

In most of the problems that follow, you will be asked to use LiveMath to graph several functions. Please do not try to print out the graphs from LiveMath. Instead, use the LiveMath graphs to help you to sketch the graphs on paper as accurately as possible by hand.

You will need to turn in a lab report consisting of all of your graphs along with answers to all of the associated questions. Please answer the questions using complete sentences with proper grammar and spelling. Your answers should be as precise and detailed as possible.

1. Let \( g(x) = x^3 - x \). For each part of this question, you will need to use LiveMath to graph four functions all in the same picture.

   a. Compare the graphs of \( y = g(x) + a \) with the graph of \( y = g(x) \) for \( a = .5, 3, \) and \(-3\).
      
      When you go to the LiveMath notebook for this lab, you will find the functions \( y = x^3 - x, \ y = x^3 - x + .5, \ y = x^3 - x + 3, \) and \( y = x^3 - x - 3 \) already graphed on the same set of axes. Sketch the four graphs on paper and then describe in as much detail as possible how the graphs compare.

   b. Compare the graphs of \( y = g(x + a) \) with the graph of \( y = g(x) \) for \( a = .5, 3, \) and \(-3\).
      
      Write out formulas for each of the four functions \( g(x), \ g(x+.5), \ g(x+3), \) and \( g(x-3) \). For example, \( g(x+3) = (x+3)^3 - (x+3) \). Then type these four functions into the LiveMath notebook in place of the functions that you looked at in part (a). Finally, sketch the four graphs on paper and then describe in as much detail as possible how the graphs compare.

   c. Compare the graphs of \( y = g(ax) \) with the graph of \( y = g(x) \) for \( a = .5, 3, \) and \( 4 \). Write out formulas for each of these four functions, enter them into LiveMath, and then describe as precisely as possible how the graphs compare.
d. Compare the graphs of \( y = f(x) \) with the graph of \( y = g(x) \) for \( a = .5, 3, \text{ and } 4 \). Once again, write out formulas for each of these four functions, enter them into LiveMath, and then describe as precisely as possible how the graphs compare.

2. a. Use LiveMath to graph \( y = \sin x \) along with all of the following four functions for \(-2\pi \leq x \leq 2\pi\):

\[
p(x) = \sin(2x) \quad q(x) = 2\sin x \quad r(x) = \sin x + 2 \quad s(x) = \sin(x + 2)
\]

Sketch of each of these functions on paper as accurately as possible, noting the period, amplitude, intercepts, and other relevant details.

b. In each case explain as precisely as possible the effect the number 2 had in modifying the graph of \( y = \sin x \).

3. Based on your answers from problems (1) and (2) and your previous knowledge of graphs of functions, answer the following questions:

a. Let \( f \) be any function and let \( a \) be any constant. Describe clearly how the graphs of \( y = f(x + a) \) and \( y = f(x) + a \) are related to the graph of \( y = f(x) \).

b. Let \( f \) be any function and let \( a \) be any positive constant. Describe clearly how the graphs of \( y = f(ax) \) and \( y = af(x) \) are related to the graph of \( y = f(x) \).

4. Let \( f(x) = \frac{2x}{3x+1} \). In this problem we will explore the effect of the absolute value function on the graph of this function. Use LiveMath to create the graphs for parts (a), (b), and (c) and then sketch the graphs on paper as accurately as possible by hand.

a. Graph the function \( f \) and discuss the main features of the graph, such as intercepts and horizontal and vertical asymptotes.

b. Graph \( y = |f(x)| \). How does the absolute value function affect the graph of \( f \)?

c. Graph \( y = f(|x|) \) and discuss the relation of this graph to the graph of \( f \).

d. In Figure 1 (at the end of this handout), the graph of a function \( y = h(x) \) is given. Sketch the graphs of \( y = h(x) \), \( y = |h(x)| \), and \( y = h(|x|) \) on the same set of axes in different colors. Explain why the graphs should look the way that you drew them.
5. Consider the quadratic polynomial \( kx^2 + (k + 1)x - (k + 2) \), where \( k \) is a constant.

   a. Graph the parabolas \( y = x^2 + 2x - 3 \) (where \( k = 1 \)) and \( y = 2x^2 + 3x - 4 \) (where \( k = 2 \)).
   Also graph the parabolas for \( k = 3, 4 \), and a few larger values. Compare the vertices and
   general shapes of these parabolas. What is happening as \( k \) increases?

   b. Notice that all of these parabolas cross the \( x \)-axis at two points. Let \( s_k \) be the larger of the
   two solutions of \( kx^2 + (k + 1)x - (k + 2) = 0 \). By looking at the graphs, approximate
   \( s_1, s_2, s_3, \) and \( s_4 \).

   c. Guess the behavior of \( s_k \) as \( k \) gets extremely large. Hint: You can always test your guess
   by graphing the polynomial for some large values of \( k \).

BONUS: Use the quadratic formula to find a formula for the value of \( s_k \) as a function of \( k \).
   Then try to use this formula to determine the behavior of \( s_k \) as \( k \) gets extremely large.

![Figure 1](image)