Here are possible examples of how the semifinal election with four candidates could work. Candidates A and B are from the first political party, and candidates C, D, and E are from the second party. Each voter ranks the candidates in preference order, and for each example we know what percentage of the people hold each preference. (There are actually $4 \times 3 \times 2 \times 1 = 24$ possible preference orderings for the four candidates, so we won’t analyze all the possibilities here.) Each voter will vote for their first choice in the semifinal election. If their first choice candidate is not available in the final election, the voter will vote for whichever surviving candidate is ranked the highest. (However, and this is a crucial point: the voters do not need to rank all four candidates in advance. They only need to determine their first choice in the semifinal election, and then they only need to determine their first choice from among the remaining candidates in the final election. Compare page ???. For purpose of this analysis, assume we know how people would rank all four candidates, even if they never have to work out this ranking themselves.)

1 Example 1: Somewhat Polarized Preference Orderings

- ABCD: 52 percent
- DCBA: 48 percent

(The line “ABCD: 52 percent” means that 52 percent of the voters have candidate A as their first choice, B as their second choice, C as their third choice, and D as their fourth choice.)

This situation would apply with an electorate that is somewhat polarized; party 1 members prefer candidate A, who is the last choice of party 2 members; and party 2 members prefer candidate D, who is the last choice of party 1 members.

result: A and D advance to final election; A is winner

2 Example 2: Slightly Polarized Preference Orderings

- BACD: 52 percent
This situation is less polarized than the preceding one: A and D are the
more extreme candidates, but the more moderate candidates B and C are
preferred by their party members.
result: B and C advance to final election; B is the winner

3 Example 3: Strategic Misrepresentation Voting Favoring a More Moderate Candidate

- ABCD: 26 percent
- BACD: 25 percent
- DCBA: 49 percent

In this situation party 1 members are nearly evenly split between candi-
date A and B, whereas party 2 members all support candidate D. If everyone
voted their true preference, A and D would advance to the final election, and
A would win 51 to 49 percent.

However, one consequence of the Arrow impossibility theorem is that
sometimes people have an incentive to strategically misrepresent their true
preferences. If party 2 members know all these numbers, they will realize
that their candidate (D) won’t win, and their last place choice A will win.
However, if 2 percent of the voters strategically misrepresent their preferences
and shift from D to B, here are the results:

- ABCD: 26 percent
- BACD: 25+2=27 percent
- DCBA: 49−2=47 percent

In this case B and D advance to the final election, and B wins 51 to
49 percent. (Nobody has an incentive to strategically misrepresent their
preferences in the two-candidate final election.)

The party 2 members who misrepresented did not succeed in electing their
favorite candidate, but they did succeed in having the winner be their third
choice (B) rather than their last choice (A). In this case the misrepresented
votes results in a more moderate candidate winning, so that might be a reasonable result. (Moderation is not always better – it depends on the policies – but electing more moderate candidates tends to have the virtue of making the losers less alienated and worried.)

Strategic voting does have its risks, which is good because we want to discourage it. Since (in this proposal) a candidate winning 55 percent of the vote in the semifinal election is automatically elected (without needing the final election), strategic voters who desert their candidate here may cost them the election. The only way to estimate these percentages in advance are by polls, and polls typically have a 3 percent margin of error. Furthermore, even if you know exactly what percentage of the votes you need to shift in the strategic misrepresentation process, there is the question of which voters will be the ones that shift. It is (and should be) illegal for people to buy or trade votes in any way, so you can’t set up arrangements to encourage only certain people to shift their vote this way. In some cases (see some of the other examples) if there are too many strategic misrepresentation voters their candidate doesn’t even make the final election.

4 Example 4: Strategic Misrepresentation Voting Favoring the Easy-to-beat Extreme Candidate

case 4: preference orderings:

- ACDE: 48 percent
- CDAE: 25 percent
- DCAE: 23 percent
- ECDA: 4 percent

In this situation, candidate A is the incumbent and is the only candidate running from party 1. This means the semifinal election has three candidates from party 2: C, D, and E. However, candidate E is a weird weak candidate, who is supported by only 4 percent of the voters and is the last choice for the other 96 percent. In particular, note that the vast majority of party 2
members rank candidate E as their last choice – even after the candidate of the other party (A). This would be the case where E’s defects as a candidate drive away even most members of his own party.

With honest voting, the result is: A and C make the final election, and then C wins the final 52 to 48 percent. Realizing this, candidate A supporters decide to strategically misrepresent their votes so that the easy-to-beat candidate E makes the final election. With the misrepresentation:

- ACDE: $48 - 22 = 26$ percent
- CDAE: 25 percent
- DCAE: 23 percent
- ECDA: $4 + 22 = 26$ percent

If 22 percent of the voters shift from A to E, the result will be that A and E make the final election, and then A will win 96 to 4 percent.

This type of strategic misrepresentation — the “support-the-easy-to-beat-candidate” misrepresentation — seems worse than the earlier kind of strategic voting. However, note that this strategy is extremely risky. If 24 percent, rather than 22 percent, shift from A to E, then A won’t even make the final election, and party 1 voters will have to choose between two party 2 candidates in the final election (which will be an easy choice, since one is the flaky candidate E). On the other hand, if only 20 percent shift, then the strategic misrepresentation strategy fails, and candidate C makes it to the final election against candidate A. Arranging for the exact correct percentage of strategic misrepresentation voters would be incredibly difficult, keeping in mind the ban on bargaining with specific voters for their votes. So, there is perhaps less need to worry about this kind of misrepresentation.

example 4b: preference orderings:

- ACDE: 40 percent
- CDAE: 21 percent
- DCAE: 20 percent
- EDCA: 19 percent
Here is an example where strategic misrepresentation voting might be the biggest problem. Candidate A is an incumbent, facing three closely-matched challengers C, D, and E from the other party. Candidate E is a rather extreme candidate who has fervent followers (19 percent of the voters), but E is the last choice of all other voters (including those who favor C and D, the other candidates from the same party).

In an honest election, A and C make it to the final election, and then C wins with 60 percent of the vote. However, the incumbent A will benefit if some voters switch to E in the semifinal:

example 4b with strategic misrepresentation:

- ACDE: 40 − 3 = 37 percent
- CDAE: 21 percent
- DCAE: 20 percent
- EDCA: 19 + 3 = 22 percent

This time A and E make it to the final election, which A then wins with 81 percent of the vote (since C and D supporters prefer A to E, the flaky candidate of their own party). Only 3 percent of the voters need to strategically misrepresent their votes in the semifinal, so it would be easier to arrange (but still it would be complicated to determine which 3 percent would do this).

*mathematical spiciness for next section: * * 2 stars*

There is a narrow window where this type of strategic misrepresentation voting will work. Here are conditions that must be met for this to happen. For situation 4b, let \( a \) represent the percentage of voters with preferences ACDE; \( c \) represent the percentage for CDAE; \( d \) represent the percentage for DCAE; and \( e \) represent the percentage for EDCA.

\( c \) must be greater than \( e \) for strategic misrepresentation to occur (otherwise A supporters don’t need to misrepresent their votes in the semifinal because they will get their wish and face \( E \) in the final election anyway.)

\( a \) must be less than 50 percent (otherwise A will win anyway and A supporters will have no need to misrepresent votes).

It is most favorable for strategic misrepresentation voting if \( c = d \). (If \( d < e \), then it’s not really correct to regard \( e \) as the fringe candidate. Therefore, we require that \( d > e \). We’ll define \( c \) to be the candidate in second
place, so by definition $c$ must be greater than or equal to $d$. The strategic misrepresentation voting becomes more likely if $c$ is smaller, and the smallest value it can have while still being greater than or equal to $d$ will be if it equals $d$ exactly.)

Since $a + c + d + e$ must equal 1, and we have $c = d$, we have:

$$a + 2c + e = 1$$

Solve for $e$:

$$e = 1 - a - 2c$$

The condition $c > e$ becomes:

$$c > 1 - a - 2c$$

$$3c > 1 - a$$

$$c > \frac{1 - a}{3}$$

$a$ can’t exceed 50 percent, as stated above, and the smaller $a$ becomes there is less opportunity for strategic misrepresentation (because $a$ voters will only desert $A$ in the semifinal primary if they know $A$ has enough votes to still make the final election anyway). The lower limit for $c$ occurs if $a$ is 50 percent, so the lower limit for $c$ is $1/6 = .1667$ $c$ cannot be less than $\frac{5}{3} = .1667$ if it is still to be greater than $e$.

Another condition on the window for strategic misrepresentation voting is that the margin by which $A$ leads $C$ must exceed the margin by which $C$ leads $E$. (It won’t do the $A$ supporters any good to misrepresent their votes if so many of them have to vote for $E$ in order for $E$ to pass $C$ that the result is that $A$ loses so many votes that $A$ falls behind $C$ – and therefore misses the final election.

$$a - c > c - e$$

Since $e = 1 - a - 2c$ (see above):

$$a - c > c - (1 - a - 2c)$$
\[ a - c > c - 1 + a + 2c \]

\[ 1 > 4c \]

\[ c < .25 \]

Therefore, strategic misrepresentation voting of this type is only possible if \( c \) is less than .25 (and, as we saw earlier, it must be greater than .167). A narrow window is good because it makes the strategic misrepresentation voting less likely.

Here is an example with \( c \) having a value slightly less than its largest possible value of .25:

- ACDE: 49.9 percent
- CDAE: 24.9 percent
- DCAE: 24.9 percent
- EDCA: 0.3 percent

With strategic misrepresentation voting:

- ACDE: 49.9 - 24.8 = 25.1 percent
- CDAE: 24.9 percent
- DCAE: 24.9 percent
- EDCA: 0.3 + 24.8 = 25.1 percent

In practice it would not be feasible to conduct strategic misrepresentation voting with this precision.

Here is an example with the value of \( c \) near its lowest bound of .1667:

- ACDE: 49.9 percent
- CDAE: 17 percent
Strategic misrepresentation voting is easy in this case because not many voters need to shift:

- **ACDE**: \(49.9 - 1 = 48.9\) percent
- **CDAE**: 17 percent
- **DCAE**: 17 percent
- **EDCA**: \(16.1 + 1 = 17.1\) percent

Note there are two factors that make the strategic misrepresentation easy in this case: (1) candidate A already has close to 50 percent of the vote, and therefore almost doesn’t need any misrepresentation; and (2) candidate E is almost as popular as candidates C and D, and so is less of a fringe candidate. However, polls have an uncertainty range of 3 to 4 percent (depending on the size of the sample) so someone planning strategic misrepresentation voting had better make sure that the margin will be at least 4 percent. This means the window is even narrower, which is good because we don’t want strategic misrepresentation voting to happen.

Another way that could defeat strategic misrepresentation voting is that when candidate D supporters learn about the devious plans of candidate A supporters, they might decide to vote for their second choice candidate C, thereby making sure that candidate C makes it to the final election and therefore wins.

Strategic misrepresentation voting is more likely to happen with a partisan primary in the current system. It is most likely when an incumbent faces no challenge for renomination, so the incumbent’s supporters feel they can interfere with the other party’s primary without having to worry that their candidate might fail to make the final ballot. The proposal described here makes misrepresentation voting less likely because voters know that if they desert their candidate in the semifinal primary they run the risk of seeing

\[\text{In 2012, there was briefly a movement for Barack Obama supporters in Wisconsin to vote for Rick Santorum in the primary because he was perceived to be a candidate that would be easier to beat. Los Angeles Times online, April 3, 2012.}\]
two candidates from the other party in the final election. With each state voting separately in the current system, there is more risk for misrepresented voters, because if the voters miscalculate and too many people vote strategically in one state primary there will still be another state primary next week, providing an opportunity to adjust tactics.

It is important to remember that, because of the Arrow theorem, no system can be designed that will be totally immune from the possibility of strategic misrepresentation voting. The proposal described here would seem to minimize the risk of this type of election distortion.

5 Example 5: Totally Deadlocked Preference Orderings

- ABCD: 25 percent
- BCDA: 25 percent
- CDAB: 25 percent
- DABC: 25 percent

This would result in four-way tie, and if somehow it was broken, there would be a two-way tie in the final election.

This case illustrates the most perverse possible preference ordering (in the sense of making it impossible to find any sensible voting system that would work). The chance of a literal tie is remote, so we won’t worry about that.

Suppose A gets 25 percent plus one vote.

- if final election opponent is B, A wins resoundingly (75 to 25).
- if final election opponent is C, result is 50 - 50.
- if final election opponent is D, A loses resoundingly (25 to 75).

(Each of the other three candidates is in the same position: they could either resoundingly win the final, resoundingly lose the final, or just about tie in the final, depending on who the other candidate is).

We have to hope that this pattern of preference orderings is unlikely – and it probably is.
6  Example 6: Eight Different Preference Patterns

Here is an example with 8 different preference patterns among the public:

- ABCD: 16 percent (more extreme party 1 voters)
- BACD: 16 percent (more moderate party 1 voters)
- CDBA: 16 percent (more moderate party 2 voters)
- DCBA: 16 percent (more extreme party 2 voters)
- CBDA: 16 percent (Independents preferring moderate candidates, leaning to party 2)
- BCAD: 16 percent (Independents preferring moderate candidates, leaning to party 1)
- ADBC: 3 percent (prefer extreme candidates to moderates, leaning to party 1)
- DACB: 1 percent (prefer extreme candidates to moderates, leaning to party 2)

The last two categories would seem to have very odd preferences – if they can’t have their first choice (the more extreme candidate of one party) then their second choice is the more extreme candidate of the other party. The results will be:

Percent of vote in semifinal:

A: 18
B: 32
C: 32
D: 18

B and C, the more moderate candidates, advance to the final election.

Percent of vote in final:

B: 32+16+3=51
C: 32+16+1=49

The voters with A as first choice all prefer B to C, and voters with D as first choice all prefer C to B. B will be the winner.
7  Examples with Three Semi-final Candidates

In some years there might only be three realistic candidates. Here are some examples.

first possibility: A and B are very similar, C is very different

<table>
<thead>
<tr>
<th></th>
<th>ABC</th>
<th>BAC</th>
<th>CAB</th>
<th>CBA</th>
<th>ACB</th>
<th>BCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>33</td>
<td>0</td>
<td>35</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

semifinal round: B and C advance;
final round: B 65 C 35; B wins

This result clearly seems most reasonable based on the preferences of the voters.

second possibility: A and C are very extreme in opposite ways; B is moderate

<table>
<thead>
<tr>
<th></th>
<th>ABC</th>
<th>ACB</th>
<th>BAC</th>
<th>BCA</th>
<th>CAB</th>
<th>CBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
<td>0</td>
<td>25</td>
<td>25</td>
<td>0</td>
<td>24</td>
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</tbody>
</table>

semifinal: B 50, A 26, C 24
final: B 74, A 26; B wins

The moderate candidate B wins.

third possibility: C is a fringe candidate.

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<thead>
<tr>
<th></th>
<th>ABC</th>
<th>ACB</th>
<th>BAC</th>
<th>BCA</th>
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<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>23</td>
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</table>
For the preferences listed here, A and B advance to the final, and then B wins the final election 53 to 47. However, the A supporters have an incentive to engage in strategic misrepresentation so that C makes the final instead of B:

\[
\begin{align*}
ABC & : 47 - 8 = 39 \\
ACB & : 0 \\
BAC & : 30 \\
BCA & : 0 \\
CAB & : 0 \\
CBA & : 23 + 8 = 31
\end{align*}
\]

With this voting pattern, A and C make the final election, and then A wins, 77 to 23 (since all of the B voters have A as their second choice). Unfortunately, this result is different than the result of the honest election mentioned above, but candidate A receives such a substantial portion of the final vote that it seems to not be a major problem. Remember, no voting system can be perfect.

8 3/7 rule for first stage election

Two other rules will apply to the first stage election in order to determine whether a quarterfinal (8-candidate) election is needed, or whether we can move directly to the semifinal (4-candidate) election: the 3/7 rule and the 2/6 rule.

Leaving aside the votes for the top candidate in the first stage election, the share of the remaining vote received by candidates 2, 3, and 4 must be greater than 3/7 (42.86 percent) in order to proceed to the semifinal election. Otherwise, the quarterfinal election with 8 candidates is held.
For example, suppose there is only one candidates from one party (candidate A), and 14 candidates from the other party (B to O), and the vote percentages in the first stage election are:

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<tr>
<td>A</td>
<td>48</td>
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<tr>
<td>B</td>
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<td>C</td>
<td>5</td>
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Excluding candidate A, the share of the vote for candidates B, C, and D is $15/52 = 28.85$, which is less than $3/7$. Since candidates B, C, and D are only slightly ahead of the other candidates, it would not seem right for candidates E to H to be eliminated at this point. It would be better to hold the quarterfinal election, with candidates A to H all participating.

In an election with 8 candidates, the total vote for the second, third, and fourth candidates is guaranteed to be at least $3/7$ of the total vote for candidates 2 to 8, so in that case the top four candidates will advance to the semifinal.

9 2/6 rule for first stage election

Here is the other rule that will apply to the first stage election in order to determine whether a quarterfinal (8-candidate) election is needed, or whether we can move directly to the semifinal (4-candidate) election.

Leaving aside the votes for the top two candidates in the first stage election, the share of the remaining vote received by candidates 3 and 4 must be greater than 2/6 (or 1/3, or 33.33 percent) in order to proceed to the
semifinal election. Otherwise, the quarterfinal election with 8 candidates is held.

For example, suppose there is an election where one party has only two candidates (A and B), and the other party has fourteen candidates (C to P), and the vote percentages in the first stage election are:

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<tbody>
<tr>
<td>A</td>
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It would not seem right for candidates C and D to advance while candidates E to P are eliminated when C and D have such a small percentage. In this case the 3/7 test is met, because the vote percent for candidates 2, 3, and 4 (33 percent) exceeds 3/7 of the vote of candidates 2 to 16 (33/73 = 45.21 percent, which is greater than 3/7 = 42.86 percent).

However, the vote for candidates 3 and 4 (10 percent) is only 10/50 = 20 percent of the total vote for candidates 3 to 16, which is less than 2/6=1/3=33.33 percent, so there will need to be a quarterfinal election with candidates A, B, C, D, E, F, G and H.

In an election with 8 candidates, the total vote for the third and fourth candidates is guaranteed to be at least 2/6 of the total vote for candidates 3 to 8, so in that case the top four candidates will advance to the semifinal.