long-run effects of government borrowing

mathematical spiciness: *** * 4 stars

Keynes correctly pointed out that requiring that the government always have a balanced budget isn’t the best idea, since that would require it to cut spending and/or raise taxes during a recession, which would tend to exacerbate the deficient spending of the recession. However, borrowing money also has a cost. People will only lend money to the government if they are promised an adequate return.

In general:

\[ G_i + r_i D_{i-1} = T_i + \Delta D_i \]

\( G_i \) is government non-interest spending in year \( i \); \( r_i \) is the interest rate in year \( i \); \( D_{i-1} \) is the level of government debt in year \( i - 1 \); \( T_i \) is government tax revenue in year \( i \); \( \Delta D_i = D_i - D_{i-1} \) is the change in government debt (the amount borrowed) in year \( i \).

Suppose the government starts with no debt in year 0, and borrows \( D_1 \) dollars in year 1:

\[ G_1 = T_1 + D_1 \]

\( (\Delta D_1 = D_1 - D_0 = D_1, \text{since there is no initial debt: } D_0 = 0) \).

\[ G_2 + r_1 D_1 = T_2 + (D_2 - D_1) \]

\[ G_2 + r_1 D_1 = T_2 + \Delta D_2 \]

\[ G_3 + r_1 D_1 + r_2 D_2 = T_3 + \Delta D_3 \]

Suppose debt has maturity of three years, so in year 4 the debt from year 1 must be repaid:

\[ G_4 + (1 + r_1) D_1 + r_2 D_2 + r_3 D_3 = T_4 + \Delta D_4 \]

In general, if \( \tau \) is the maturity period of the debt:

\[ G_k + (1 + r_{k-\tau}) \Delta D_{k-\tau} + r_{k-\tau+1} \Delta D_{k-\tau+1} + r_{k-\tau+2} \Delta D_{k-\tau+2} + \ldots r_{k-1} \Delta D_{k-1} = T_k + \Delta D_k \]
Here are some special cases.

Case A: one time debt, repaid three years in future:

\[ G_1 = T_1 + D_1 \]

\[ G_2 + r_1 D_1 = T_2 \]

\[ G_3 + r_1 D_1 = T_3 \]

\[ G_4 + (1 + r_1)D_1 = T_4 \]

In year 1, spending can be higher than taxes because of the borrowing. In year 2 and 3, spending must be lower than taxes because some revenue is needed to pay interest. In year 4, spending must be significantly lower than taxes, because both the interest and the principal amount of the debt from year 1 must be repaid. However, after that society continues on with no more debt. The ability of the government to spend money in years 2, 3, and 4 is limited by the need to repay debt, but that doesn’t necessarily mean society will be poorer in those years. The big question is: what was the borrowed money in year 1 used for? If borrowed money is used to finance projects with long-term benefits, then the borrowing is probably a good idea (it depends on how the cost compares to the benefits). If the borrowed money finances current spending, then it is much harder to make the case that the borrowing is justified. The same rule applies to a person: borrowing for a house likely is beneficial; borrowing for a party is less likely to be beneficial.

Case B: one time debt that is never paid off:

\[ G_1 = T_1 + D_1 \]

\[ G_2 + r_1 D_1 = T_2 \]

\[ G_3 + r_1 D_1 = T_3 \]

\[ G_4 + (1 + r_4)D_1 = T_4 + D_1 \]
Note: in year 4, the government has to borrow the same amount it originally borrowed \((D_1)\), in order to repay the principle on that amount when it comes due.

We can cancel out a \(D_1\) on the left and right side of the above equation, so it becomes:

\[
G_4 + r_4 D_1 = T_4
\]

And it continues just like that:

\[
G_5 + r_4 D_1 = T_5
\]

\[
G_6 + r_4 D_1 = T_6
\]

and so on. Note that the result is a permanent diversion of tax revenue to pay the interest on the initial debt. In this example we have imagined that the principal amount comes due every three years, which means that the interest rate might change every three years (but no more often than that). One risk of doing this is that future interest rates will be higher, which increases the amount of tax revenue that must be diverted to pay the interest.

Case C: do one time borrowing in year 1, but instead of using tax revenue to pay the interest, continue borrowing each year to repay the previous interest (also repeat the warning: don’t try this at home).

\[
G_1 = T_1 + D_1
\]

\[
G_2 + r_1 D_1 = T_2 + r_1 \Delta D_1
\]

\[
\Delta D_2 = r_1 \Delta D_1
\]

\[
G_3 + r_1 D_1 + r_2 r_1 \Delta D_1 = T_3 + r_1 D_1 + r_2 r_1 \Delta D_1
\]

... suppose interest rates stay the same:

\[
G_k + (r_1 + r_1^2 + r_1^3 + \ldots + r_1^k) \Delta D_1 = T_k + (r_1 + r_1^2 + r_1^3 + \ldots + r_1^k) \Delta D_1
\]
In this case the government non-interest spending equals tax revenue each year, but the debt continually grows because none of it is ever paid back. Therefore, the amount that needs to be borrowed each year continually grows.

If the government is going to keep growing debt, it either needs to encourage greater wealth supply (either domestically or internationally) with higher interest rates, or it needs to discourage private wealth demand, so the government can take over a greater share of wealth (by crowding out private investment).