1.  
\[ P = 41.667 - .611Q \]
\[ TR = PQ = (41.667 - .611Q)Q = 41.667Q - .611Q^2 \]
\[ MR = \frac{dTR}{dQ} = 41.667 - 2 \times .611Q = 41.667 - 1.222Q \]

set the derivative equal to zero:
\[ Q = \frac{41.667}{1.222} = 34.097 \]
\[ P = 41.667 - .611 \times 34.097 = 20.83 \]

2.  
\[ TC = 200 + 100Q + 4.5Q^2 \]
\[ MC = \frac{dTC}{dQ} = 100 + 9Q \]
\[ P = 400 - 3Q \]
\[ TR = 400Q - 3Q^2 \]
\[ MR = \frac{dTR}{dQ} = 400 - 6Q \]

set \( MR = MC \):
\[ 100 + 9Q = 400 - 6Q \]
\[ 15Q = 300 \]
\[ Q = 20 \quad P = 340 \]

3.  
\[ TC = .8Q^3 - 10Q^2 + 48Q + 64 \]
\[ MC = \frac{dTC}{dQ} = 2.4Q^2 - 20Q + 48 \]

set marginal cost equal to price:
\[ 2.4Q^2 - 20Q + 48 = 60 \]
\[ 2.4Q^2 - 20Q - 12 = 0 \]
\[ Q = \frac{20 \pm \sqrt{(-20)^2 - 4 \times 2.4 \times (-12)}}{2 \times 2.4} = \frac{20 \pm \sqrt{515.2}}{4.8} = 8.895 \]

4.  
\[ T = gx + ky + kx + ky \]

Solve for \( y \):
\[ 2ky = T - gx - kx = T - (g + k)x \]
\[ y = \frac{T}{2k} - g + k \frac{x}{2k} \]
\[ A = xy = \frac{Tx}{2k} - g + k \frac{x^2}{2k} \]
\[ \frac{dA}{dx} = \frac{T}{2k} - g + k \frac{x}{k} \]

set the derivative equal to zero:
\[ \frac{T}{2k} - g + k \frac{x}{k} = 0 \]
\[ \frac{T}{2k} = g + k \frac{x}{k} \]
\[ x = \frac{T}{2(g + k)} \]
\[ y = \frac{T}{2k} - g + k \left( \frac{T}{2(g + k)} \right) = \frac{T}{2k} - \frac{T}{4k} = \frac{T}{4k} \]

if \( g = k \):
\[ x = \frac{T}{4k} \]

if \( g = 2k \):
\[ x = \frac{T}{6k} \]

5.
\[ S = 2x^2 + 3xy \]
\[ y = \frac{S - 2x^2}{3x} = s \frac{x^{-1}}{3} - \frac{2}{3} x \]
\[ V = x^2 y = x^2 \left( s \frac{x^{-1}}{3} - \frac{2}{3} x \right) = s \frac{x}{3} - \frac{2}{3} x^3 \]
\[ \frac{dV}{dx} = s \frac{3}{3} - 2x^2 \]

set the derivative equal to zero:
\[ 0 = s \frac{3}{3} - 2x^2 \]
\[ s \frac{3}{3} = 2x^2 \]
\[ s \frac{3}{6} = x^2 \]
\[ x = \sqrt{s} \frac{3}{6} \]
\[ y = \frac{s - 2x^2}{3x} = \frac{s - 2 \times (\frac{3}{6})}{3 \sqrt{s} \frac{3}{6}} \]
6.

\[ y = 2x^3 - 93x^2 + 1008x + 204 \]

\[ \frac{dy}{dx} = y' = 6x^2 - 186x + 1008 \]

\[ \frac{d^2y}{dx^2} = y'' = 12x - 186 \]

set the derivative equal to zero:

\[ 0 = 6x^2 - 186x + 1008 \]

Use the quadratic formula:

\[ x = \frac{-(-186) \pm \sqrt{(-186)^2 - 4 \times 6 \times 1008}}{2 \times 6} \]

\[ x = \frac{186 \pm \sqrt{186^2 - 4 \times 6 \times 1008}}{2 \times 6} \]

\[ x = \frac{186 \pm \sqrt{34,596 - 24,192}}{12} \]

\[ x = \frac{186 \pm \sqrt{10,404}}{12} \]

\[ x = \frac{186 \pm 102}{12} \]

There are two values of \( x \): \( x = 24 \) and \( x = 7 \).

When \( x = 24 \), the second derivative \( \frac{d^2y}{dx^2} = 12 \times 24 - 186 = 102 \). Since this is positive, the curve is concave up at this point, so \( x = 24 \) represents a local minimum.

When \( x = 7 \), the second derivative \( \frac{d^2y}{dx^2} = 12 \times 7 - 186 = -102 \). Since this is negative, the curve is concave down at this point, so \( x = 7 \) represents a local maximum.
7. 
\[ h = -\frac{1}{2}gt^2 + v_0t \]

(a) \( \frac{dh}{dt} = -gt + v_0 = -9.8t + 12 \)

(b) Set the derivative equal to zero:

\[ 0 = -gt + v_0 \]

Solve for \( t \):

\[ gt = v_0 \]

\[ t = \frac{v_0}{g} = \frac{12}{9.8} = 1.22 \text{ seconds} \]

(c) Insert the formula for \( t \) at the time of maximum height into the formula for height:

\[ h_{\text{max}} = -\frac{1}{2}g \left( \frac{v_0}{g} \right)^2 + v_0 \left( \frac{v_0}{g} \right) \]

\[ h_{\text{max}} = -\frac{1}{2} \frac{v_0^2}{g} + \frac{v_0^2}{g} \]

\[ h_{\text{max}} = \frac{1}{2} \frac{v_0^2}{g} = \frac{12^2}{2 \times 9.8} = 7.35 \text{ meters} \]

(d) Set \( h \) equal to zero:

\[ 0 = -\frac{1}{2}gt^2 + v_0t \]

Solve for \( t \):

\[ \frac{1}{2}gt^2 = v_0t \]

One solution will be where \( t = 0 \). This means that the height of the ball is zero at the instant it is thrown. To find the other value for \( t \), divide both sides by \( t \):

\[ \frac{1}{2}gt = v_0 \]

Solve for \( t \):

\[ t = \frac{2v_0}{g} = 2.44 \text{ seconds} \]

Note that the time until the ball hits the ground is twice as long as the time for it to reach its highest point.

\[ (e) \frac{d^2h}{dt^2} = -g \]
8.

sparkle detergent: \[ Q_1 = 200 - 20P_1 + 6P_2 \]
dazzle detergent: \[ Q_2 = 30 - 10P_2 + 4P_1 \]

\[ Z = (P_1 - 5)(200 - 20P_1 + 6P_2) + (P_2 - 2)(30 + 4P_1 - 10P_2) \]

\[
egin{array}{cccc}
200P_1 & -20P_1^2 & +6P_1P_2 & -1000 \\
+100P_1 & +4P_1^2 & -60 & +30P_2 \\
-8P_1 & & +20P_2 & \\
\end{array}
\]

The profit function is:

\[ Z = 292P_1 - 20P_1^2 + 10P_1P_2 - 1060 + 20P_2 - 10P_2^2 \]

Find the partial derivatives:

\[ Z_{P_1} = \frac{\partial Z}{\partial P_1} = 292 - 40P_1 + 10P_2 = 0 \]
\[ Z_{P_2} = \frac{\partial Z}{\partial P_2} = 10P_1 + 20 - 20P_2 = 0 \]

Write the equation system as:

\[-40P_1 + 10P_2 = -292 \]
\[10P_1 - 20P_2 = -20 \]

Divide the second equation by two. The new equation system is:

\[-40P_1 + 10P_2 = -292 \]
\[5P_1 - 10P_2 = -10 \]

Add the two equations together:

\[-35P_1 = -302 \]
\[P_1 = \frac{-302}{-35} = 8.629 \quad \text{round to 8.63} \]

Put this value into the second equation to find \( P_2 \):

\[ 5 \times 8.629 - 10P_2 = -10 \]
\[43.15 + 10 = 10P_2 \]
\[P_2 = \frac{53.15}{10} = 5.315 \quad \text{round to 5.32} \]

To double check, put the value for \( P_1 \) into the first equation:

\[-40 \times 8.629 + 10P_2 = -292 \]
\[10P_2 = 345.16 - 292 = 53.16 \]
\[P_2 = \frac{53.16}{10} = 5.316 \quad \text{round to 5.32} \]