MAT 3751: Real Analysis II

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University and Departmental Mission: Seattle Pacific University seeks to be a premier Christian university fully committed to engaging the culture and changing the world by graduating people of competence and character, becoming people of wisdom, and modeling grace-filled community. The mathematics department at Seattle Pacific University seeks to provide excellent instruction to enable our students to be competent in the mathematics required for their chosen fields, and to share our expertise with the community through service and leadership. Hence, common goals for students in mathematics courses include 1) becoming competent in the topics covered in the course, 2) demonstrating skills and attitudes which contribute to professional, ethical behavior, 3) the ability to communicate mathematically, in both written and verbal form, and 4) learning to appreciate the beauty and utility of mathematics.

Course Overview: *What is analysis?* Analysis is the branch of mathematics that deals with properties of functions. Real analysis deals primarily with real-valued functions of real numbers; essentially it is the theory behind elementary calculus. We'll begin by wrapping up some topics from MAT 3749, including topology of the real line, limits of sequences of real numbers, and properties of those limits. These topics lay the foundation for the rest of the course, which involves a rigorous study of functions of a real variable, including limits, continuity, derivatives, and integrals.

The course should help to sharpen your analytical and critical reasoning skills and improve your ability to express mathematical ideas with clarity and coherence. By the end of the course, you should attain a greater appreciation of the need for precise definitions, careful reasoning, and close argument in mathematics. You should also have an increased familiarity with and understanding of many topics first encountered in calculus, including basic properties of sequences, series, limits, continuity, and derivatives.

Learning Objectives: By the end of the course, a student will be able to:

- State and apply the $\varepsilon - N$ definition of the limit of a sequence and state properties associated with convergence and divergence of sequences.
- State and apply the following theorems associated with convergence of sequences:
  - the algebraic and order limit theorems;
  - the monotone convergence theorem;
  - the Cauchy criterion; and
  - the Bolzano-Weierstrass theorem for sequences.
- State and apply the following definitions and theorems:
  - the $\varepsilon - \delta$ definitions of continuity and the limit of a function;
  - the intermediate value theorem;
  - the formal definition of a derivative;
  - the mean value theorem;
  - L'Hospital's Rule;
  - Taylor's Theorem;
  - the definition of a Riemann integral; and
  - the fundamental theorem of calculus.
- Write formal proofs making use of each of the topics above.
- Independently read, comprehend, and summarize material in a mathematics textbook.

Prerequisites: Grade of C- or higher in MAT 3749 (Introduction to Analysis) prior to starting this course.

Textbooks: *Analysis with an Introduction to Proof*, Stephen R. Lay, 4th edition, Pearson/Prentice Hall, 2001.
Exploratory Examples for Real Analysis, Joanne E. Snow & Kirk E. Weller, MAA, 2003.

Software: The course will include some lab activities which require the use of the mathematical software package Maple. The software is available for your use in all of the open computer labs on campus. If you wish to use the software on your own computer, a student license for Maple can be purchased for about $75 (details will be provided in class).

Grading and Course Expectations

Attendance: Attendance will not be taken, but if you expect to succeed in this course, it is essential that you come to class every day. Unless you have an acceptable excuse ***and*** make special arrangements with me ***before*** class begins, missing an exam or quiz or failing to turn in an assignment on time will result in a grade of zero. Late homework will not be accepted for any reason, but if you have an acceptable excuse and contact me before class, I will drop the homework score.

NOTE: Things such as oversleeping, lack of preparation, or sneezing twice are NOT acceptable excuses. Acceptable excuses include a death in your immediate family or a *severe* illness, and *you are responsible for providing me with documentation of your excuse.*

Homework: The only way to truly learn mathematics is to work as many exercises as possible. You cannot learn to do mathematics by watching someone else do mathematics or listening to someone else talk about mathematics – you must actually do it yourself and practice, practice, practice. Furthermore, the material each class period builds on what came before, so it is essential to keep up with the work every day. As a result, there will be homework assignments given virtually every class period which will often be collected at the next class meeting.

General homework policies:

- Homework must be turned in by 12:30 (not 12:31) on the day it is due; late homework will not be accepted for any reason. If you have an excused absence and make arrangements with me *before* class starts, the homework score will be dropped.
- You are strongly encouraged to come to my office to ask me questions about the homework.
- You are also encouraged to work with other students on the homework, but unless otherwise indicated for a particular assignment, you must individually write up and turn in your own solutions.
- You are also required to list on your paper all other individuals that you worked with or that gave you assistance with the homework – failure to do so will be considered cheating (turning in someone else's work as your own).
- Homework must be neat and easily readable or you will receive NO credit.
- You must show all of your work–a correct answer with no justification will also be worth NO credit.
- Your lowest homework score will be dropped from your final grade.

Reading Assignments: You will regularly be asked to read sections in the textbook and respond to several brief questions about the section. These responses will be sent to me by e-mail, and will be due the night before class meets.

Quizzes: There will be brief quizzes approximately once a week which will cover definitions, statements of theorems, and basic ideas from the course.

Exploratory Labs: There will be several labs during the course which ask you to explore basic concepts of real analysis. You will work on each lab with a small group of other students, and each group will prepare a written report on the lab. Most of the labs will require the use of computers and Maple.

Exams: There will be two exams, a midterm and a final. Both exams will be given in two parts: an in-class portion and a take-home portion. The in-class midterm is tentatively scheduled for Monday, February 8. The in-class final exam will be cumulative and will be given during the regularly scheduled final exam period (10:30-12:30 on Wednesday, March 17).

Course Grades: Grades will be determined by a weighted average of homework and labs (30%), quizzes (10%), reading assignments (10%), the midterm (20%), and the final exam (30%). Course grades will be based on the following scale:

93-100% A	90-92% A-	
87-89% B+	83-86% B	80-82% B-
77-79% C+	73-76% C	70-72% C-
67-69% D+	60-66% D	Below 60% E

A grade of I (incomplete) is only given for non-academic reasons such as a severe illness that prevents you from completing the course. You must have a passing grade on the material that you have completed in order to receive an incomplete.

Academic Dishonesty: The current edition of the SPU Undergraduate Catalog describes the University's commitment to academic integrity, which is breached by academic dishonesty of various kinds. Examples of academic dishonesty include copying another's work on an exam, bringing concealed answers to an exam, turning in another person's work as your own, committing plagiarism, assisting another student in cheating, or lying to the instructor. The *minimum* penalty for academic dishonesty in any form will be a zero for the assignment or exam in question; in severe cases, academic dishonesty will result in a failing grade for the course. In addition, all students have an obligation to make efforts to prevent other students from cheating and to report incidents of cheating or plagiarism.

Office Hours: My office hours will be announced in class during the first week of class and will be posted on the course web pages and outside my office door. You are strongly encouraged to drop by my office to ask questions, discuss problems, and just to get to know me better. If you are unable to meet with me during my scheduled office hours, I am available at other times by appointment. I also maintain an "open door" policy at my office – any time that my door is open you are welcome to drop in to talk to me, even if it is not during my scheduled office hours. *Please note that I also work as the statistician for a research lab at the University of Washington, so I am generally not available on campus on Thursdays. Please plan accordingly.*

Additional Notes

E-mail: All SPU students have an SPU e-mail address. I will occasionally make use of these SPU e-mail addresses to send information to all members of the class, so you should check your e-mail regularly. If you do not use your SPU e-mail account, there is a utility available through Banner to set up your SPU e-mail account to forward messages to some other e-mail address. I strongly recommend doing this so that you do not miss any important messages.

Please note that while it can be a great tool for quick communication (such as scheduling an appointment to talk with me face-to-face), *e-mail is rarely a good substitute for face-to face conversations and is very poorly suited for answering mathematical questions*. When you come to my office to ask me questions, I engage you in a discussion about the problem, ask questions about what ideas you have for approaching the problem, explore various possible approaches (and what goes wrong with some of them), etc. In the process, I can usually find out precisely where your difficulties lie and help you to learn how to get past them. Such a conversation is impossible by e-mail. Furthermore, typing and e-mailing mathematical symbols is very time consuming, and the resulting equations in the e-mail e-mails often come out garbled (or even completely missing).

Students with Disabilities: In accordance with Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990, students with specific disabilities that qualify for academic accommodations need to contact Disabled Student Services in the Center for Learning. Disabled Student Services in turn will send a Disability Verification Letter to me indicating what accommodations have been approved.

Once you have done this, you should also make an appointment to meet with me as early as possible in the quarter to discuss the details of how we will implement the accommodations in this course.

Inclement Weather: SPU maintains an Emergency Closure Hotline (206-281-2800). In the event of inclement weather or an emergency that might close the university, please call the Hotline for the most up-to-date closure information or check the SPU website. Both will be updated before 6:00 a.m.

Emergency Procedure: Please note the emergency procedures posted in the classroom and note all emergency exits. In case of an emergency (fire, earthquake, hazardous material spillage, bomb threat, etc.), the class will evacuate the building and gather in the Alumni Center parking lot outside Otto Miller Hall. Please try to stay together so that we can check that everyone has made it safely out of the building.

Tentative Course Schedule

The table below provides a tentative listing of topics that I plan to cover from the textbook. The exact dates on which we cover material will probably vary somewhat from this list. Also, topics may be added to or removed from this list at any time.

Date	Topics Covered
1/6	Brief review: the completeness axiom and limits of sequences (Sections 12, 16, 17) Exploratory Lab: Conditions Related to Convergence
1/8	Limit Theorems (Section 17)
1/11	Monotone Sequences and Cauchy Sequences (Section 18)
1/13	Exploratory Lab: Understanding the Limit Superior and Limit Inferior
1/15	Exploratory Lab: Continuity and Sequences
1/18	**No class** – Martin Luther King Jr. Birthday
1/20	Compact Sets (Section 14)
1/22	Subsequences (Section 19)
1/25	Wrap-up of sequences
1/27	Exploratory Lab: Definitions of Continuity and the Limit of a Function
1/29	Limits of Functions (Section 20)
2/1	Lab Activity: Experience with the $\varepsilon - \delta$ Definitions of Continuity and Limit
2/3	Continuous Functions (Section 21)
2/5	Review/catch-up
2/8	**Midterm Exam**
2/10	Properties of continuous functions (Section 22)
2/12	Uniform continuity (Section 23)
2/15	**No class** – Presidents' Day
2/17	Introduction to the derivative (Section 25)
2/19	More on derivatives
2/22	The mean value theorem (Section 26)
2/24	L'Hospital's Rule (Section 27)
2/26	Taylor's Theorem (Section 28)
3/1	The Riemann Integral (Section 29)
3/3	More on integrals
3/5	Properties of the Riemann Integral (Section 30)
3/8	The Fundamental Theorem of Calculus (Section 31)
3/10	Catch-up day
3/12	Review/wrap-up
3/17	**Final Exam** 10:30-12:30

Modifications to the course requirements can be made at any time.
It is *your* responsibility to know all course requirements as described here or announced in class.