MAT 3749 Homework #10

Turn in exercises 7.12, 7.15ad, 7.16, 7.17, 7.25, 7.29, and 7.30 from Section 7 of the textbook. Note that there is a typo in the text on exercise 7.12; it should say $(f+g)(y) = f(y) + g(y)$, not $f(y) + f(y)$.

In addition, prove each of the following:

1. Let *A*, *B*, and *C* be sets, and let $f : A \to B$ and $g : B \to C$.
   a. Suppose that if $g \circ f : A \to C$ is surjective.
      i. Is *f* necessarily surjective? (Prove it or give a counterexample.)
      ii. Is *g* necessarily surjective? (Prove it or give a counterexample.)
   b. Give an example to show that $g \circ f : A \to C$ can be bijective even if neither *f* nor *g* is bijective.
   c. If both *f* and $g \circ f$ are bijective, then is *g* necessarily bijective? (Prove it or give a counterexample.)

2. Let $n \in \mathbb{Z}$ and let $g : \mathbb{Z} \to \mathbb{N} \cup \{0\}$ be defined by

$$g(n) = \begin{cases} 2n & \text{if } n \geq 0 \\ -2n-1 & \text{if } n < 0 \end{cases}.$$

   a. Evaluate $g(n)$ for $n = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.
   b. Prove that if $x \in \mathbb{Z}$, then $g(x) \in \mathbb{N} \cup \{0\}$.
   c. Prove that *g* is injective.
   d. Prove that *g* is surjective.
   e. Find g^{-1}.