MAT 3749: Introduction to Analysis

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University and Departmental Mission: Seattle Pacific University seeks to be a premier Christian university fully committed to engaging the culture and changing the world by graduating people of competence and character, becoming people of wisdom, and modeling grace-filled community. The mathematics department at Seattle Pacific University seeks to provide excellent instruction to enable our students to be competent in the mathematics required for their chosen fields, and to share our expertise with the community through service and leadership. Hence, common goals for students in mathematics courses include 1) becoming competent in the topics covered in the course, 2) demonstrating skills and attitudes which contribute to professional, ethical behavior, 3) the ability to communicate mathematically, in both written and verbal form, and 4) learning to appreciate the beauty and utility of mathematics.

Course Overview: This course was designed to meet two major goals of the mathematics curriculum. First, it will provide an introduction to the methods and practice of mathematical proof and a foundation for further mathematical study. Second, it will provide an introduction to the topics of analysis: functions, cardinality, topology of the real line, and limits. The first half of this course will emphasize logical arguments, the writing of proofs, and communication in mathematics. Unlike Calculus and most other lower division mathematics courses, much of higher mathematics has more to do with logical reasoning than with calculation. The goals of the first part of this course are (i) to introduce you to some ideas, terminology, and habits of thought that are used throughout higher mathematics, and (ii) to develop your skills in constructing logical arguments and solving problems that require reasoning rather than calculation.

The emphasis of the rest of the course will be on real numbers and the foundations of real analysis. We will define and construct the set of real numbers and study various properties of sets of real numbers. In addition, we will learn the precise, formal definition of a limit of a sequence of real numbers and study properties of these limits. These topics lay the foundation for a rigorous study of functions of a real variable, including limits, continuity, derivatives, and integrals, which are covered in detail in MAT 3751 (Real Analysis II).

What is analysis? Analysis is the branch of mathematics that deals with properties of functions. *Real analysis* deals primarily with real-valued functions of real numbers; essentially it is the theory behind elementary calculus.

Learning Objectives: By the end of the course, you should

- ❑ understand and be able to apply the basic rules of logic which form the foundation of mathematical proof;
- ❑ know and be able to effectively use the basic principles and methods of logical proof, including methods of direct proof, proof by contrapositive, proof by contradiction, mathematical induction, and case analysis;
- ❑ be able to disprove incorrect mathematical statements through the use of concrete counterexamples;
- ❑ understand and be able to apply basic terminology and properties associated with each of the following areas:
  - o algebra of sets
  - o relations (especially equivalence relations)
  - o functions (including injections, surjections, and bijections)
  - o cardinality of sets; countable and uncountable sets
- ❑ know and be able to work with basic properties of the real number system, including:
  - o field axioms, order axioms, and the completeness axiom
  - o basic topology of the real line, including open and closed sets and compactness
- ❑ develop familiarity with constructions of natural numbers, integers, rational numbers, and real numbers;
- ❑ understand the concept of the limit; in particular, you should
  - o know and be able to apply the formal definition of a limit of a sequence;
  - o understand the historical need for replacing intuitive notions of limits with precise modern definitions;
  - o be familiar with bounded sequences, monotone sequences, subsequences, Cauchy sequences, and related theorems.

In addition to the specific content oriented objectives above, this course will help you to

- ❑ sharpen your analytical and critical reasoning skills and improve your ability to express mathematical ideas both orally and in writing with clarity and coherence;
- ❑ attain a greater appreciation of the need for precise definitions, careful reasoning, and close argument in mathematics;
- ❑ learn to read and understand proofs written by others and recognize invalid proofs;
- ❑ learn to transfer mathematical ideas from intuitions and conjectures to formal arguments and proofs on paper;
- ❑ prepare for further upper division mathematics courses and greater abstraction; and
- ❑ have fun accomplishing these objectives, even if the material is difficult and takes a lot of time and effort.

Prerequisites: Successful completion of MAT 2720 (Discrete Mathematics). In addition, MAT 1228 (Series & Differential Equations) and 2401 (Linear Algebra) are strongly recommended.

Textbook: *Analysis with an Introduction to Proof*, Stephen R. Lay, 4th edition, Pearson/Prentice Hall, 2001.
I also recommend *The Nuts and Bolts of Proof*, Antonella Cupillari, 3rd edition, Elsevier/Academic Press.

Software: The course will include some lab activities which require the use of the mathematical software package Maple. The software is available for your use in all of the open computer labs on campus. If you wish to use the software on your own computer, a student license for Maple can be purchased for about $75 (details will be provided in class).

Grading and Course Expectations

Attendance: Attendance will not be taken, but if you expect to succeed in this course, it is essential that you come to class every day. Unless you have an acceptable excuse ***and*** make special arrangements with me ***before*** class begins, missing an exam or quiz or failing to turn in an assignment on time will result in a grade of zero. Late homework will not be accepted for any reason, but if you have an acceptable excuse and contact me before class, I will drop the homework score.
NOTE: Things such as oversleeping, lack of preparation, or sneezing twice are NOT acceptable excuses. Acceptable excuses include a death in your immediate family or a *severe* illness, and *you are responsible for providing me with documentation of your excuse.*

Homework: The only way to truly learn mathematics is to work as many exercises as possible. There will be homework assignments given virtually every class period which will be due at the next class meeting. Homework must be turned in by the time class starts on the day it is due; late homework will not be accepted for any reason. However, your lowest homework score will be dropped from your final grade.
You are strongly encouraged to come to my office to ask me questions about the homework. You are also encouraged to work with other students on the homework, but unless otherwise indicated for a particular assignment, you must individually write up and turn in your own solutions. You are also required to list on your paper any other individuals that you worked with or that gave you assistance with the homework – failure to do so will be considered cheating (turning in someone else's work as your own). Homework must be neat and easily readable or you will receive NO credit. You must show all of your work–a correct answer with no justification will also be worth NO credit.

Quizzes: There will be brief quizzes approximately once a week which will cover definitions, statements of theorems, and basic ideas from the course.

Exploratory Labs: There will be several labs during the course which ask you to explore basic concepts of real analysis. You will work on each lab with a small group of other students, and each group will prepare a written report on the lab. Some of the labs will require the use of computers and Maple.

Exams: There will be two exams, a midterm and a final. Both exams will be given in two parts: an in-class portion and a take-home portion. The in-class midterm is tentatively scheduled for Friday, November 7. The in-class final exam will be cumulative and will be given during the regularly scheduled final exam period (10:30-12:30 on Wednesday, December 10).

Course Grades: Grades will be determined by a weighted average of homework and quizzes (40% of the overall grade), the midterm (25%), and the final exam (35%). Course grades will be based on the following scale:

	93-100% A	90-92% A-
87-89% B+	83-86% B	80-82% B-
77-79% C+	73-76% C	70-72% C-
67-69% D+	60-66% D	Below 60% E

A grade of I (incomplete) is only given for non-academic reasons such as a severe illness that prevents you from completing the course. You must have a passing grade on the material that you have completed in order to receive an incomplete.

Academic Dishonesty: Academic dishonesty includes copying another's work on an exam, preparing for an exam by using test questions from a stolen exam, bringing concealed answers to an exam, turning in another person's work as your own, committing plagiarism, assisting another student in cheating, or lying to the instructor. The *minimum* penalty for cheating or plagiarism in any form will be a zero for the assignment or exam in question. In addition, all students have an obligation to make efforts to prevent other students from cheating and to report incidents of cheating or plagiarism. Further details regarding SPU's academic dishonesty policies can be found online in the SPU Undergraduate Catalog. (http://www.spu.edu/acad/UGCatalog/20089/GeneralInfo/policies.asp).

Office Hours: My office hours will be announced in class during the first week of class and will be posted on the course web pages and outside my office door. You are strongly encouraged to drop by my office to ask questions, discuss problems, and just to get to know me better. If you are unable to meet with me during my scheduled office hours, I am available at other times by appointment. I also maintain an "open door" policy at my office – any time that my door is open you are welcome to drop in to talk to me, even if it is not during my scheduled office hours. *Please note that I also work as the statistician for a research lab at the University of Washington, so I am not available on campus on Tuesdays and Thursdays. Please plan accordingly.*

Additional Notes

E-mail: All SPU students have an SPU e-mail address. I will occasionally make use of these SPU e-mail addresses to send information to all members of the class, so you should check your e-mail regularly. If you do not use your SPU e-mail account, there is a utility available through Banner to set up your SPU e-mail account to forward messages to some other e-mail address. I strongly recommend doing this so that you do not miss any important messages.

Please note that while it can be a great tool for quick communication (such as scheduling an appointment to talk with me face-to-face), *e-mail is rarely a good substitute for face-to face conversations and is very poorly suited for answering mathematical questions*. When you come to my office to ask me questions, I engage you in a discussion about the problem, ask questions about what ideas you have for approaching the problem, explore various possible approaches (and what goes wrong with some of them), etc. In the process, I can usually find out precisely where your difficulties lie and help you to learn how to get past them. Such a conversation is impossible by e-mail. Furthermore, typing and e-mailing mathematical symbols is very time consuming, and the resulting equations in the e-mail e-mails often come out garbled (or even completely missing).

Students with Disabilities: Students with disabilities need to contact Disabled Student Services in the Center for Learning to request academic accommodations. Disabled Student Services sends letters out to all your professors indicating the appropriate accommodations for the classroom based on your disability.

Once you have done this, you should also make an appointment to meet with me as soon as possible to discuss the details of how we will implement the accommodations in this course.

Inclement Weather: SPU maintains an Emergency Closure Hotline (206-281-2800). In the event of inclement weather or an emergency that might close the university, please call the Hotline for the most up-to-date closure information or check the SPU website. Both will be updated before 6:00 a.m.

Emergency Procedure: Please note the emergency procedures posted in the classroom and note all emergency exits. In case of an emergency (fire, earthquake, hazardous material spillage, bomb threat, etc.), the class will evacuate the building and gather in the Alumni Center parking lot outside Otto Miller Hall. Please try to stay together so that we can check that everyone has made it safely out of the building.

Tentative Course Schedule

The table below provides a tentative listing of topics that I plan to cover from the textbook. The exact dates on which we cover material will almost certainly vary somewhat from this list. Also, topics may be added to or removed from this list at any time.

Date	Topics Covered
9/29	Introduction to the course Logical Connectives (Lay Section 1)
10/1	Quantifiers (Section 2)
10/3	Techniques of Proof I (Section 3) (basic concepts, direct proof, proof by contrapositive, disproving by counterexample)
10/6	Techniques of Proof II (Section 4) (proofs involving quantifiers, proof by contradiction, case analysis)
10/8	More on proof writing
10/10	More on proof writing
10/13	Basic Set Operations (Section 5)
10/15	**No class:** day of common learning
10/17	Relations (Section 6)
10/20	Equivalence Relations (Section 6)
10/22	Functions (Section 7)
10/24	More on Functions
10/27	Natural Numbers and Induction (Section 10)
10/29	Cardinality (Section 8)
10/31	More on cardinality **Take-home midterm distributed**
11/3	Axioms for Set Theory (Section 9)
11/5	Constructing natural numbers, integers, and rational numbers from basic set theory
11/7	**In-class midterm exam; Take-home midterm due by 11:00**
11/10	Ordered Fields (Section 11)
11/12	The Completeness Axiom (Section 12)
11/14	Topology of the Reals (Section 13)
11/17	More on the completeness axiom, plus brief coverage of compactness (Section 14)
11/19	Sequences: Convergence (Section 16)
11/21	More on limits
11/24	Sequences: Limit Theorems (Section 17)
11/26	Monotone Sequences and Cauchy Sequences (Section 18)
11/28	**No class** – Thanksgiving break
12/1	Subsequences (Section 19)
12/3	Limits of Functions (Section 20) **Take-home final exam distributed**
12/5	Introduction to derivatives and integrals; preview of MAT 3751
12/8	Review and wrap-up
12/10	**In-class Final Exam** 10:30-12:30; Take-home final exam due by 10:30

Modifications to the course requirements can be made at any time.
It is *your* responsibility to know all course requirements as described here or announced in class.