Probability Theory

MAT 2375 Autumn 2007

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University and Departmental Mission: Seattle Pacific University seeks to be a premier Christian university fully committed to engaging the culture and changing the world by graduating people of competence and character, becoming people of wisdom, and modeling grace-filled community. The mathematics department at Seattle Pacific University seeks to provide excellent instruction to enable our students to be competent in the mathematics required for their chosen fields, and to share our expertise with the community through service and leadership. Hence, common goals for students in mathematics courses include 1) becoming competent in the topics covered in the course, 2) demonstrating skills and attitudes which contribute to professional, ethical behavior, 3) the ability to communicate mathematically, in both written and verbal form, and 4) learning to appreciate the beauty and utility of mathematics.

Course Overview: This course is an introduction to the mathematical study of randomness and uncertainty with an emphasis on their application to statistics and data analysis. The concept of probability represents the connection between the unpredictability of a single event in a random process and the predictability and uniformity of multiple events. Probability theory allows us to make very accurate predictions about the *pattern* of variation of a random process *in the long term*. Topics in this course will include basic rules of probability, combinatorial methods, independence and conditional probability, discrete and continuous probability distributions, expected values, moment generating functions, and an introduction the statistical notions of significance, testing, and estimation. In addition, we will look at applications of probability to a wide variety of fields.

Class time will be split between lecture and small group activities designed to lead you to discover concepts and explore properties of probability and statistics. You will be asked to work through these activities in class with a partner. I strongly encourage you to work collaboratively with other students, as I firmly believe that you will be able to help one another with your learning and that talking about the ideas helps to solidify everyone's understanding of the material.

To whet your appetite for the types of questions that we will be investigating in this course, I present this sampling of problems:

- If a hospital were to distribute a group of four newborn babies to their mothers at random, what is the probability that none of the mothers will get the correct baby? How many babies would we "expect" to get paired with the correct mother? What if there were 10 babies? What if there were 100?
- If you test positive for the AIDS virus, what is the probability that you have actually been infected?
- What is the probability that two people in a group share the same birthday? How many people would there have to be in the room before you would expect there to be at least a 50-50 chance of finding at least two of them with birthdays on the same day?
- If the defense in a criminal trial introduces results of a polygraph test that are favorable to the defendant, what is the probability that the defendant is actually telling the truth? What impact should this evidence have on the verdict? Should such evidence even be admissible?
- When an individual threatens to commit suicide by jumping from a high structure, they are often jeered by crowds and people passing by. Are crowds more likely to jeer during the summer than the rest of the year?
- A much higher percentage of men than of women was admitted to the graduate school of the University of California at Berkeley in the fall quarter of 1973. Can we attribute the difference to sex discrimination?

Learning Objectives: By the end of the course, you should acquire:

- an understanding of the fundamental concepts of the mathematical theory of probability, including conditional probability, independence, random variables, probability distributions, and expectation;
- the ability to perform a wide variety of probability calculations and derivations;
- an understanding of some fundamental statistical concepts related to probability, such as significance, randomization, testing, and estimation;
- the ability to use a computer to analyze data and to conduct simulations to solve probability problems;
- the ability to use probabilistic reasoning to approach and solve practical problems from a variety of fields;

In addition to the specific content oriented objectives above, you should

- be able to prove basic theorems of probability;
- be able to provide written explanations of the ideas behind key concepts from the course;
- have improved skills at problem solving and critical thinking;
- have improved skills at solving complex, multi-step problems; and
- have fun accomplishing these objectives, even if the material is difficult and takes a lot of time and effort.

Course Materials:

Textbook and Class Handouts: The primary materials used for the course will be a collection of handouts given out in class. In addition, there will be a custom text available from the SPU bookstore consisting of selected materials from *Investigating Statistical Concepts, Applications, and Methods* by Beth Chance and Allan Rossman.

Computer Software: We will make use of both Microsoft Excel and the statistical software package Minitab to perform calculations, to create graphics for analyzing data, and to conduct simulations to approximate long-run behavior of random phenomena. Minitab is available for your use on all computers in labs on campus. If you wish to use Minitab on your own computer, you can purchase a license at www.e-academy.com/minitab. The cost is $29.99 for a license which expires after 6 months, $49.99 for a 12-month license, or $99.99 for a permanent license. Minitab will also be used in MAT 2376 in the winter quarter.

Grading and Course Expectations

Attendance: Attendance will not be taken, but if you expect to succeed in this course, it is essential that you come to class every day. Unless you have an acceptable excuse ***and*** make special arrangements with me ***before*** class begins, missing an exam or failing to turn in an assignment on time will result in a grade of zero. Late homework will not be accepted for any reason, but if you have an acceptable excuse and contact me before class, I will drop the homework score.

NOTE: Things such as oversleeping, lack of preparation, or sneezing twice are NOT acceptable excuses. Acceptable excuses include a death in your immediate family or a *severe* illness, and *you are responsible for providing me with documentation of your excuse.*

Homework: The only way to truly learn mathematics is to work as many problems as possible. There will be homework assignments given virtually every class period which will be due at the next class meeting. Homework must be turned in by the time class starts on the day it is due; late homework will not be accepted for any reason. However, your lowest homework score will be dropped from your final grade.

You are strongly encouraged to come to my office to ask me questions about the homework. You are also encouraged to work with other students on the homework, but unless otherwise indicated for a particular assignment, you must individually write up and turn in your own solutions. You are also required to list on your paper any other individuals that you worked with or that gave you assistance with the homework – failure to do so will be considered cheating (turning in someone else's work as your own). Homework must be neat and easily readable or you will receive NO credit. You must show all of your work–a correct answer with no justification will also be worth NO credit.

Exams: There will be two exams, a midterm and a final. The midterm is tentatively scheduled for Wednesday, October 31. The cumulative final exam will be given in two parts. The first part will be take-home, and the second part will be in-class during the regularly scheduled final exam period (8:00-10:00 on Thursday, December 8). Most of the questions on the in-class exams will be *very* similar to exercises from the homework. You also may be expected to be able to state some definitions and to provide proofs of certain key theorems from the course.

Course Grades: Homework assignments will be worth 40% of your course grade, the midterm exam will be worth 25%, and the final exam will count for 35% of the overall grade. Course grades will be based on the following scale:

93-100% A	90-92% A-	
87-89% B+	83-86% B	80-82% B-
77-79% C+	73-76% C	70-72% C-
67-69% D+	60-66% D	Below 60% E

A grade of I (incomplete) is only given for non-academic reasons such as a severe illness that prevents you from completing the course. You must have a passing grade on the material that you have completed in order to receive an incomplete.

Academic Dishonesty: Academic dishonesty includes copying another's work on an exam, preparing for an exam by using test questions from a stolen exam, bringing concealed answers to an exam, turning in another person's work as your own, committing plagiarism, or assisting another student in cheating. The *minimum* penalty for cheating or plagiarism in any form will be a zero for the assignment or exam in question. In addition, all students have an obligation to make efforts to prevent other students from cheating and to report incidents of cheating or plagiarism. Further details regarding SPU's academic dishonesty policies can be found online in the SPU Undergraduate Catalog (http://www.spu.edu/acad/UGCatalog/20078/GeneralInfo/policies.asp).

Office Hours: My office hours will be announced in class during the first week of class and will be posted on the course web pages and outside my office door. You are strongly encouraged to drop by my office to ask questions, discuss problems, and just to get to know me better. If you are unable to meet with me during my scheduled office hours, I am available at other times by appointment. I also maintain an "open door" policy at my office – any time that my door is open you are welcome to drop in to talk to me, even if it is not during my scheduled office hours. *Please note that I also work as the statistician for a research lab at the University of Washington, so I am not available on campus on Tuesdays and Thursdays. Please plan accordingly.*

Additional Notes:

Prerequisites: Mastery of the material from MAT 1225 and 1226 (Calculus I and II) is required. Specifically, we will use techniques of differentiation and integration, improper integrals, integration by parts, L'Hopital's rule, and double integrals. While it is not absolutely essential, MAT 1228 (Series & Differential Equations) is also *strongly* recommended as a prerequisite because infinite series will occasionally be used in this class. It is possible to complete the course without any knowledge of infinite series, but it will make it much more difficult to attain a full understanding of certain topics.

Calculators: Calculators will be permitted on all exams. Any basic scientific calculator or a graphing calculator such as the TI-82, TI-83, or TI-85 will be allowed. However, calculators such as the TI-89 or TI-92 which are capable of symbolic manipulation will NOT be permitted. If you have any doubt as to whether or not your calculator is acceptable, please ask me as soon as possible.

E-mail: All SPU students have an SPU e-mail address. I will occasionally make use of these SPU e-mail addresses to send information to all members of the class, so you should check your e-mail regularly. If you do not use your SPU e-mail account, there is a utility available through Banner to set up your SPU e-mail account to forward messages to some other e-mail address. I strongly recommend doing this so that you do not miss any important messages.

Please note that while e-mail can be a great tool for quick communication, *it is rarely a good substitute for face-to face conversations and is very poorly suited for answering mathematical questions.* When you come to my office to ask me questions, I engage you in a discussion about the problem, ask questions about what ideas you have for approaching the problem, explore various possible approaches (and what goes wrong with some of them), etc. In the process, I can usually find out precisely where your difficulties lie and help you to learn how to get past them. Such a conversation is impossible by e-mail. Furthermore, typing and e-mailing mathematical symbols is very time consuming, and the resulting equations in the e-mail often come out garbled (or even completely missing).

Students with Disabilities: Students with disabilities need to contact Disabled Student Services in the Center for Learning to request academic accommodations. Disabled Student Services sends letters out to all your professors indicating the appropriate accommodations for the classroom based on your disability. Once you have done this, you also must make an appointment to meet with me as soon as possible to discuss the details of how we will implement the accommodations in this course.

"Probability is the very guide of life."
– Cicero

"Everything existing in the universe is the fruit of chance."
– Democritus

"Chance is perhaps a pseudonym of God when he does not wish to sign his work."
– Anatole France

"You believe in a God who plays dice, and I in complete law and order."
– Albert Einstein in a letter to Max Born

"The conception of chance enters into the very first steps of scientific activity in virtue of the fact that no observation is absolutely correct. I think chance is a more fundamental conception than causality; for whether in a concrete case, a cause-effect relation holds or not can only be judged by applying the laws of chance to the observation."
– Max Born

**"God knows I am no friend of probability theory.
I have hated it from the first moment our dear friend Max Born gave it birth"**
– Erwin Schrödinger to Albert Einstein

"How dare we speak of the laws of chance? Is not chance the antithesis of law?"
– Bertrand Russell

"I find that the harder I work, the more luck I seem to have."
– Thomas Jefferson

"Luck? In my experience there's no such thing."
– Obi-Wan Kenobi

Tentative Class Schedule

The table below provides a very tentative schedule of topics to be covered in class. The exact dates on which material will be covered WILL vary somewhat from this list. Topics may be added to or removed from this list at any time.

Date	Material to be Covered (chapter numbers are from class handouts)
9/24	Chapter 1: Introduction to Probability Probability as long-term relative frequency Case study: Random babies (Investigation 1.6.1 from Chance/Rossman) Expected value
9/26	Chapter 1 (cont.) Estimating probability by computer simulation Introduction to the use of Minitab
10/1	Chapter 1 (cont.) Probability and statistics: Introduction to tests of statistical significance (1.5 from Chance/Rossman)
10/3	Chapter 2: Basic Probability Rules Notation of set theory Basic probability rules
10/8	Chapter 2 (cont) More on basic probability rules
10/10	Chapter 2 (cont.) Conditional probability, independence, multiplication rules, & the law of total probability
10/15	Chapter 2 (cont.) Axioms of probability and proofs of basic probability rules
10/17	Chapter 3: Learning to Count Counting methods: permutations, combinations, and hypergeometric probabilities
10/22	Chapter 3 (cont.) Fisher's exact test; introduction to tests of significance (1.7.1 from Rossman/Chance); The birthday problem
10/24	**No Class – Day of Common Learning**
10/29	Review
10/31	**Midterm Exam**
11/5	Chapter 4: Bayes' Theorem Case studies: AIDS testing and forensic evidence
11/7	Chapter 5: Discrete Random Variables The p.m.f and the c.d.f.
11/12	**No Class – Veteran's Day**
11/14	Chapter 5 (cont.) Mathematical expectation: the mean, variance, and standard deviation Examples: uniform distribution, hypergeometric distribution
11/19	Chapter 5 (cont.) More examples: Bernoulli trials, the binomial, negative binomial, and geometric distributions
11/21	Chapter 6: Continuous Random Variables The p.d.f. c.d.f, expected value, and variance Examples: the uniform and exponential distributions
11/26	Chapter 6 (cont) Examples: the uniform and exponential distributions
11/28	Catch-up day! (There's no way I'll be on schedule by now. ☺)
12/3	Review
12/6	**Final Exam – 8:00 to 10:00**

Modifications to the course requirements can be made at any time. It is *your* responsibility to know all course requirements as described here or announced in class.